be obtained by impressing a stamp only when $w \geqslant 1.5$. In order to impress a stamp of given form to the required depth we must, for $w = 1.5; 2; 2.5$ apply the forces per unit width of the strip, equal to $5.58\tau_s h$, $5.9\tau_s h$, $6.22\tau_s h$.

REFERENCES

1. KRAVCHUK A.S., On the Hertz problem for linearly and non-linearly elastic bodies of finite size. PMM, 41, 2, 1977.
2. KUZ'MENKO V.I., Contact problems of the theory of plasticity for complex loading. PMM, 48, 3, 1984.
3. LIONS J.-L. and MAGENES E., Non-homogeneous Boundary Value Problems and Their Applications. Berlin, N.Y. Springer-Verlag, 1972.
4. KUZ'MENKO V.I., On the unloading process for contact interaction. PMM, 49, 3, 1985.
5. KANTOROVICH L.V. and AKILOV G.P., Functional Analysis. Moscow, Nauka, 1977.
6. KOITER V.T., General Theorems of the Theory of Elastoplastic Media. Moscow, Izd-vo inostr. lit. 1961.

Translated by L.K.

PMM U.S.S.R.,Vol.50,No.3,pp.367-372,1986
Printed in Great Britain

0021-8928/86 $10.00+0.00

GENERALIZED SOLUTIONS IN THE THEORY OF PLASTICITY*

B.A. DRUYANOV

Conditions prevailing on the surfaces of the strong velocity discontinuities in rigid-plastic media were studied by many workers, e.g. /1, 2/. However, in all cases known to the author the conditions were obtained by utilizing a passage to the limit, when the surface of the discontinuity was considered as a limit to which a layer tends, the layer undergoing an intense deformation and its thickness tending to zero. Meanwhile, it is desirable to obtain the conditions at the discontinuities by intrinsic means from the system of equations itself, without bringing in the irrelevant concepts on what represents the surface of the discontinuity. To this end the equations must be given in divergent form. In the theory of plasticity the main difficulties in this respect are encountered in connection with the law of flow and the law controlling the hardening.

The present paper shows that certain generalization of the Mises principle makes it possible to impart to the inequality expressing it a divergent form and enables us to write it in integral form. From this it follows that in the incompressible plastic medium the surface of discontinuity in the tangential velocity component serves as the surface of maximum tangential stresses, with tangential stress directed along the velocity jump vector. In a compressible plastic medium the stress discontinuity is determined from the condition that the direction of the six-dimensional deformation velocity "vector" is continuous. We note that the integral form of the Mises inequality was used in /3/ to prove the existence and uniqueness of the solution. It was not, however, given in divergent form, and the conditions at the discontinuities were not considered.

With regard to the equation describing the hardening law, it can be reduced to divergent form when the specific plastic work is used as the hardening parameter.

The problem considered here is that of steady motion of a strip of finite thickness undergoing pure shear, in a rigid-plastic hardening medium. The emission of heat caused by plastic deformation and its effect

**Prikl.Matem.Mekhan.*,50,3,483-489,1986

on the functions of state and the forces of inertia are all taken into account. It is shown that under certain specified conditions the strip thickness may tend to zero, i.e. the appearance of isothermal velocity discontinuities is possible. The adiabatic and quasistatic cases are discussed. It is noted that the insufficiency of the continuous solutions in the connected perfect thermorigid-plastic medium was discussed in /4/.

In important practical problems the thickness of the layer undergoing pure shear is small compared with the characteristic dimension and can be neglected. Therefore the discontinuous solutions can also be brought in in the general case when heat conduction is taken into account. In this case the condition of temperature continuity should be omitted.

Some of the results of this paper were given in /5/.

1. Model of the material. We will construct a model of the material following, basically, /6/. The flow of the medium is governed by the following equations:

$$\sigma_{ij,j} = \rho v_i^{\cdot} \tag{1.1}$$

$$\Phi(\tau_{ij}, \theta, \chi) = 0 \quad (\tau_{ij} = \sigma_{ij} - \sigma\delta_{ij},\ \sigma = \sigma_{ij}\delta_{ij}/3) \tag{1.2}$$

$$\varepsilon_{ij} = \lambda\Phi_{ij} \quad (\varepsilon_{ij} = (v_{i,j} + v_{j,i})/2,\ \Phi_{ij} = \partial\Phi/\partial\sigma_{ij}) \tag{1.3}$$

$$\chi^{\cdot} = a_{ij}(\tau_{ij}, \theta)\,\varepsilon_{ij} \quad (a_{ij} = a_{ji}) \tag{1.4}$$

$$\rho e^{\cdot} = -q_{i,i} + \sigma_{ij}\varepsilon_{ij} \tag{1.5}$$

Here (1.1) are the equations of motion, ($\boldsymbol{\sigma}$ is the stress tensor, ρ is the density, $\mathbf{v}$ is the velocity, $\mathbf{v}^{\cdot} = d\mathbf{v}/dt$), (1.2) is the yield condition, (χ is the hardening parameter, θ is the temperture, (1.3) is the associated flow law, (1.4) is the hardening law, (1.5) is the equation of heat flow ($e = e(\theta, \chi)$ is the internal energy per unit mass and $\mathbf{q}$ is the heat flux vector).

The Fourier law of thermal conduction must be generalized for the hardening plastic medium, taking the parameter χ into account. We shall write it in the form

$$q_i = -\varkappa(\theta, \chi)\,\theta_{,i} \tag{1.6}$$

Physical considerations show that the thermal conductivity $\varkappa$ decreases as χ /7/ increases.

We complete the description of the model by specifying the dissipation of mechanical energy

$$D = \sigma_{ij}\varepsilon_{ij} - \rho\eta\chi^{\cdot} \tag{1.7}$$

Here $\eta = \eta(\theta, \chi)$ is the thermodynamic force corresponding to the hardening parameter χ. The term $\eta\chi^{\cdot}$ reflects the fact that not all plastic work is converted into heat. Part of this work, equal to $\rho\eta\chi^{\cdot}$, is stored at microlevels and is reversible. The part is small and does not exceed 20% of $\sigma_{ij}\varepsilon_{ij}$ /6, 7/.

If entropy can be introduced for the medium of the type discussed here, then e, η and s (the entropy per unit mass) can be expressed in terms of the free energy per unit mass $f(\theta, \chi)$ /6/

$$\eta = \partial f/\partial\chi, \quad s = \partial f/\partial\theta, \quad e = f + \theta s$$

In this case η and e cannot be specified arbitrarily, since they are connected by the relation

$$\frac{\partial}{\partial\theta}\left(\frac{\eta}{\theta}\right) + \frac{1}{\theta^2}\frac{\partial e}{\partial\chi} = 0 \tag{1.8}$$

2. Reduction of the fundamental equation to divergent form. The plastic potential (1.2) is independent of σ, and therefore the equations of plastic flow (1.3) yield the equation of incompressibility

$$v_{i,i} = 0 \tag{2.1}$$

Using (2.1) we can write the equations of motion and the equation for the law of conservation of energy in the divergent form

$$\sigma_{ij,j} = \rho v_{i,0} + p_{,i} \tag{2.2}$$

$$\rho e_{,0} + p_{,0} + (q_i + (\rho e + p)\,v_i)_{,i} - (\sigma_{ij}v_i)_{,j} = 0 \tag{2.3}$$

$$p = \rho v_i v_i/2, \quad v_{i,0} = \partial v_i/\partial t$$

The hardening condition can be written in divergent form in the case when the specific plastic work is taken as the hardening parameter, i.e. when the equation of hardening law has the form $\chi^{\cdot} = \sigma_{ij}\varepsilon_{ij}$. Using the equations of motion, we can write this relation in the form

$$\chi_{,0} - p_{,0} + ((\chi + p)\,v_i)_{,i} = (\sigma_{ij}v_i)_{,j} \tag{2.4}$$

In the case of a compressible plastic medium we use the plastic work per unit mass

$\dot{\chi} = \rho\sigma_{ij}e_{ij}$ as the condition of hardening. Using the equations of motion and law of conservation of mass, we can also write this relation in divergent form /8/.

Let us pause and consider the relations of the associated flow law (1.3). We know that the law is equivalent to the Mises maximum principle, which can be expressed by the inequality

$$(\sigma_{ij}-\sigma_{ij}^*)\,\varepsilon_{ij}\geqslant 0 \tag{2.5}$$

Here ε is the real deformation rate tensor, σ is the real stress tensor and σ^* is the stress tensor satisfying, at the point M of the body under discussion, the yield inequality for the given θ, χ

$$\Phi(\tau_{ij}^*, \theta, \chi)\leqslant 0 \tag{2.6}$$

Henceforth, we shall assume that a solution of the initial-boundary value problem in question exists in class C_1, i.e. continuously differentiable functions σ_{ij}, v_i, θ and χ exist which we shall call real, and which satisfy the initial and boundary conditions, all equations of the model, and in particular conditions (2.5) and (2.6).

Let us consider the class of functions σ_{ij}, continuously differentiable in some neighbourhood ω' of an arbitrary point M of the body, and satisfying in this neighbourhood the yield inequality (2.6) for real θ, χ and the equations of motion (2.2) with real right-hand sides. We shall denote this class of functions by Σ_1, noting that the real σ_{ij} belong to Σ_1.

We shall now require that the stresses $\sigma_{ij}, \sigma_{ij}^*$ in the Mises inequality belong to Σ_1. Such a contraction of the class of admissible σ_{ij} does not reduce the generality of the Mises principle, and does not affect the procedure for obtaining the associated flow law (1.6) from inequality (2.5), since the dependence of σ_{ij} on the coordinates is not taken into account in this procedure. Indeed, according to the Mises principle, in the classical approach the real stress tensor σ imparts, at some point of the body, a maximum to the function $\sigma_{ij}e_{ij}$ for real e_{ij}, out of all σ_{ij}, satisfying the yield inequality (2.6) at this point, for real θ and χ. Therefore, here we have the problem of a conditional extremum of the functions $\sigma_{ij}e_{ij}$, in which σ_{ij} appear as arguments. It is because of that that the dependence of σ_{ij} on the coordinates is of no importance. Constructing the Lagrange function $\varphi = \sigma_{ij}e_{ij} - \lambda\Phi$ (λ is the Lagrange multiplier) we obtain (1.3). Thus the equations of associated flow law (1.3) follow from (2.5) also when $\sigma_{ij}\in\Sigma_1$.

Let us return to condition (2.5). Using relations (2.1) and (2.2), we can write it in divergent form ($\mathbf{v}$ is the real velocity)

$$((\sigma_{ij}-\sigma_{ij}^*)\,v_i)_{,j}\geqslant 0 \tag{2.7}$$

Integrating (2.7) over an arbitrary neighbourhood $\omega\subset\omega'$ of the point M and passing to the surface integral, we obtain (s is the boundary of ω, ν in the outer normal)

$$\int\limits_s (\sigma_{ij}-\sigma_{ij}^*)\,v_i\nu_j\,ds\geqslant 0 \tag{2.8}$$

By virtue of the arbitrariness of ω and the reversibility of the operations used, we find that when $\sigma_{ij}, \sigma_{ij}^*\in\Sigma_1$ and $v_1\in C_1$, conditions (2.5) and (1.3) follow from (2.8). We find, however, that (2.8) retains its meaning also when $\sigma_{ij}, \sigma_{ij}^*, v_i$ are discontinuous.

It can be shown that inequality (2.8) also remains valid in the case of compressible plastic media, i.e. when the yield condition depends on the mean stress /8/.

Let us now introduce the class Σ of functions $\sigma_{ij}, \sigma_{ij}^*$ which may become discontinuous on some surfaces separating the body into a finite number of parts in every one of which σ_{ij}, v_i belongs to Σ_1. When $\sigma_{ij}, v_i\in\Sigma$, inequality (2.5) is inapplicable. Let us replace it by (2.8), i.e. let us assume that inequality (2.8) must hold at all points of the body including points on the surface of discontinuity. Thus we shall regard (2.8) as a generalization of the Mises inequality to the case of discontinuous σ_{ij} and v_i.

The divergent form of the relations (1.6), (2.1)-(2.4), (2.8) enables us to rewrite them in integral form, admitting of discontinuities of the tangential velocity and stress components, in the normal heat flux components, and in the hardening parameter χ.

3. Relations at the surface of strong discontinuities. Let us apply inequality (2.8) to the region $\omega\subset\omega'$ containing the point M of the surface of discontinuity Γ (Fig.1). Let us pass to the limit as $h\to 0$. The quantities $s_{ij}=\sigma_{ij}-\sigma_{ij}^*$ satisfy the equations of equilibrium and are therefore continuous on Γ. Therefore (2.8) leads, in the limit, to the relation $s_{ij}\nu_j[v_i]\geqslant 0$ or (square brackets denote a jump)

$$\sigma_{ij}E_{ij}\geqslant\sigma_{ij}^*E_{ij}\quad (E_{ij}=([v_i]\,\nu_j+[v_j]\,\nu_i)) \tag{3.1}$$

Thus the real stress tensor σ imparts, on each side of the surface of discontinuity, a maximum to the expression $\sigma_{ij}E_{ij}$ out of all σ_{ij} satisfying the yield inequality. We note that according to the Mises principle the tensor $\mathbf{E}$ should be regarded as real. Solving the problem of the conditional extremum we find that the stresses on both sides of the surface of velocity discontinuity must satisfy the relations

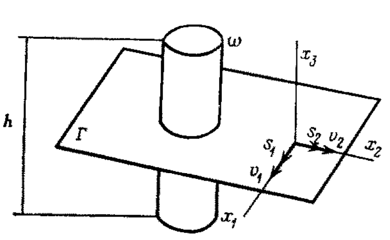

Fig.1

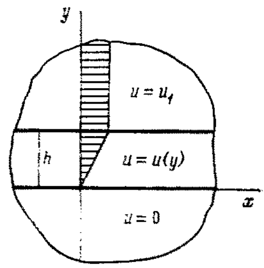

Fig.2

$$E_{ij} = \Lambda\Phi_{ij} \tag{3.2}$$

(Λ is an undefined function of a point on the surface of the velocity discontinuity).

If the yield surface is convex, relations (3.2) together with the yield condition determine, for the given E_{ij}, the stresses on each side of the surface of discontinuity. If on the other hand the yield surface contains a flat part, then the stresses on both sides of the surface of discontinuity will not be given uniquely by Eqs.(3.2).

Relations (3.2) show that at points on the surface of discontinuity, the tensor σ, regarded as a six-dimensional vector, corresponds on both sides to the point of the yield surface at which the "vector" $\mathbf{E}$ is directed along the normal to it. Since the vector ε is also directed along the normal to the yield surface on both sides of Γ, it follows that the vector ε does not change its direction during passage across the surface of the velocity discontinuity.

All the above arguments, including relations (3.1) and (3.2), refer not only to incompressible, but also to compressible media /8/. In the case of an incompressible medium, the velocity component normal to the surface of discontinuity is continuous on this surface.

Let us rewrite condition (3.1) in the form (τ_1 and τ_2 are tangential stresses at the surface of the discontinuity)

$$\tau_1 [v_1] + \tau_2 [v_2] \geqslant \tau_1^* [v_1] + \tau_2^* [v_2] \tag{3.3}$$

Inequality (3.3) means that real τ_1 and τ_2 must impart a maximum to the stress $\tau_1 [v_1] + \tau_2 [v_2]$ out of all tangential stresses satisfying the yield condition. From this it follows that the "stress vector" tangent to the surface of discontinuity on each of its sides, must be directed along the velocity jump vector, and that is must be equal in magnitude to the limiting tangential stress k. Thus strong velocity jumps are possible only on the surface of maximum shear.

Let us consider the hardening law (2.4). In the associated coordinate system the equation leads to the expression (v is the rate of propagation of the discontinuity, and u is the projection of the material velocity on the direction of the velocity jump vector)

$$v [\chi] = [ku] - \rho v [u^2/2] \tag{3.4}$$

The equations of motion, taking $[v_n] = 0$ into account, yield

$$[\sigma_n] = 0, \quad [k] = \rho v [u] \tag{3.5}$$

The heat influx equation yields (q_n is the projection of the heat flux vector on the normal to the surface of discontinuity) /9/

$$\rho v [e] = [q_n] + [\chi] \tag{3.6}$$

Using (3.5), we can write relation (3.4) in the form

$$2\rho v^2 [\chi] = [k^2] \tag{3.7}$$

The Fourier law of heat conduction implies that the temperature is continuous. In the adiabatic case ($\varkappa = 0$) however, the temperature may have discontinuities.

Let the form and position of the surface of velocity discontinuity in the body be given at some instant of time. We shall also assume that the tangential velocity jump $[u]$ is also given on this surface. Then from (3.5) and (3.7) we determine the rate of propagation of the discontinuity of v and $[\chi]$, and from (3.6) $-$ $[q_n]$. We should remember that the maximum tangential stress k is regarded as the parameter of the material, hence the dependence of k on θ and χ is given.

Thus if $[\theta] = 0$, then $[u]$ and $[\chi]$ are connected by Eq.(3.5). In the adiabatic case $q_i = 0$ and $[q_n] = 0$, but $[\theta] \neq 0$. Eqs.(3.5), (3.6) and (3.7) define $[\theta]$, $[\chi]$ and v.

The conditions at the jump are obtained, in the case of the quasistatic flow, from (3.4) and (3.5), with $\rho = 0$. The conditions show that in this case $[k] = 0$.

If $[\theta] = 0$, then we obtain $[\chi] = 0$ and $[u] = 0$. Therefore, in a quasistatic flow no discontinuities are possible. This also applies to isothermal flow /2/.

If on the other hand $[\theta] \neq 0$ (the adiabatic case), then $[\chi]$ and $[\theta]$ are connected by the relation $[k] = 0$. The latter relation together with (3.4) and (3.6), which in the case have the form $v[\chi] = k[u]$, $\rho v[e] = [\chi]$, determines v, $[\chi]$, $[\theta]$.

Restricting ourselves to piecewise smooth surfaces of discontinuity, we shall find the generalized solution. As usual, we shall understand by the generalized solution the solution satisfying the differential equations of the model in the domains of continuity of the solution vector and the conditions on the jumps at the surface of discontinuity. In the case of a divergent system of equations, such a solution satisfies the system of equations in integral form.

4. The problem of pure shear in a layer of finite thickness. As was shown before, the surface of strong discontinuity in the medium in question must coincide with the surface of maximum shear. Therefore, in modelling the structure of the discontinuity it is natural to assume that shear deformations prevail. In this connection we shall consider steady plane flow of the medium under pure shear in a layer of thickness h (Fig.2). Let $v = \text{const}$ be the velocity of the material in the direction of the axis Oy, $u = u(y)$ in the direction of the axis Ox, $u = 0$ when $y \leqslant 0$, $u = u(y)$ for $0 \leqslant y \leqslant h$, $u = u_1 = \text{const}$ for $y \geqslant 0$, $u(0) = 0$, $u(h) = u_1$. This implies that the regions $y < 0$ and $y > h$ do not become deformed.

The equations of motion, of hardening and of heat flow (under the Fourier law of heat conduction) in this case have the form

$$\frac{dk}{dy} = \rho v \frac{du}{dy}, \quad v\frac{d\chi}{dy} = a\frac{du}{dy}, \quad \rho v\frac{de}{dy} = \frac{d}{dy}\left(\varkappa \frac{d\theta}{dy}\right) + k\frac{du}{dy} \tag{4.1}$$

($k(\theta, \chi)$, $a(\theta, \chi)$ are assumed to be known functions of their arguments, k is the shear yield point and $a = a_{13}$ is the hardening function appearing in (1.4).

In order to determine u, θ, χ for $0 \leqslant y \leqslant h$, we pose the following boundary conditions: $u = \theta = \chi = 0$, $d\theta/dy = \theta_0'$ when $y = 0$; $u = u_1$ when $y = h$ (the quantity h is to be determined).

The first two equations of (4.1) yield

$$(\rho v^2 - \beta a) d\chi/d\theta = \alpha a, \quad u = \rho v(k - k_0) \tag{4.2}$$
$$\alpha = \partial k/\partial\theta, \quad \beta = \partial k/\partial\chi > 0, \ k_0 = k(0, 0)$$

We see from this that if $\rho v^2 \neq \beta a$, for $0 \leqslant y \leqslant h$, then there exists a unique solution of this system $\chi = \chi(\theta)$, $u = u(\theta)$, satisfying the condition $u = \chi = 0$ for $\theta = 0$, and, that inverse functions also exist. The condition $\rho v^2 \neq \beta a$ means that the velocity v must not be equal to the velocity of propagation of the weak discontinuities $v_* = (a\beta/\rho)^{1/2}$ for $0 \leqslant y \leqslant h$. We note that system (4.1) is integrable in quadratures in many important cases, for example when $a = k$, i.e. when plastic work is taken as the hardening parameter. From (4.1) we obtain

$$y = I(\theta), \quad I(\theta) = \int_0^\theta \left[\rho v(e - e_0) + \varkappa\theta_0' - \frac{k^2 - k_0^2}{2\rho v}\right]^{-1} \varkappa\, d\theta \tag{4.3}$$

The solution obtained will be real when the specific plastic power ku' is non-negative (we neglect the term $\rho\eta\chi^{\cdot}$). The latter condition reduces, in the present case, to the inequality $\chi' \geqslant 0$, or to

$$\frac{d\chi}{dy} = \frac{\alpha a\left(2\rho v^2(e - e_0) + 2\rho v\varkappa\theta_0' - (k^2 - k_0^2)\right)}{2v\varkappa\rho^2(v^2 - v_*^2)} \geqslant 0$$

In some neighbourhood of the point $\theta = \chi = 0$ the numerator has the same sign as θ_0', therefore when $\theta_0' > 0$, the inequality will hold provided that $v > v_*$ and when $\theta_0' < 0$, provided that $v < v_*$. From (4.2) and (4.3) we obtain

$$h = I(\theta_1), \quad u_1 = \rho v(k_1 - k_0), \quad \chi_1 = \chi(\theta_1) \tag{4.4}$$

If U_1 is given, then the above relation yields θ_1, χ_1, h.

Let the function $v_*(\theta, \chi)$ be such, that $v_* = v_0 = \text{const}$ when $\theta = 0$ and χ is arbitrary. For example, when $a = k = (k_0^2 + 2b(\theta)\chi - C(\chi)\theta^m)^{1/2}$, $m > 1$, we have $v_* = ((b - 0{,}5C'\theta^m)/\rho)^{1/2}$, which for $\theta = 0$ we obtain $v_*(0, 0) = (b(0)/\rho)^{1/2} = \text{const}$. In this case when $v = v_0$, the first equation of (4.2) has a solution $\theta = 0$. Let us now pass to the limit as $v \to v_0$. The solutions of the first equation of (4.2) depend continuously on v, therefore when $v \to v_0$, we find that $\theta \to 0$ for any $y \in [0, h]$. Moreover, we see from (4.4) that $h \to 0$, i.e. the deformed layer becomes a surface of discontinuity.

Thus we can have temperatures at which jumps in isothermal velocity can occur.

We see from (4.1) that in the adiabatic formulation of the problem only a discontinuous solution exists.

5. On discontinuous solutions in a heat conducting medium. It has been shown that in general a continuous solution exists of the problem of the structure of the discontinuity in a heat conducting medium. We know that the introduction of discontinuities simplifies the solution considerably. Computations using the formulas of Sect.4 show that the thickness of the deformed layer in the problem of the structure of the discontinuity is small in the case of real materials compared with the characteristic dimensions of the instrument and the blanks, if we have in mind technological problems. The thickness can therefore be neglected in many cases. This is obviousy equivalent to a refusal to consider the Fourier Eq.(1.6). From this it follows that discontinuous soltuions can be brought in also when heat conduction is taken into account, although in this case the condition $[\theta] = 0$ must be omitted. The magnitude of the jump in θ together with $[\chi]$ and $[q_n]$ is determined from the relations (3.4)—(3.6), and the rate of propagation of the discontinuity is arbitrary.

This approach was used in /10/ for the case of quasistatic flows.

Sect.4 was written with help of E.A. Svyatova.

REFERENCES

1. HILL R., Discontinuity relations in mechanics of solids. In: Progress in Solid Mechanics, 2, 1961.
2. IVLEV D.D. and BYKOVTSEV G.I., Theory of a Selfhardening Plastic Body. Moscow, Nauka, 1971.
3. DUVAUT G. and LIONS J.L., Inequalities in Mechanics and Physics. Berlin, Springer-Verlag, 1976. 1976.
4. KAMENYARZH YA.A., On some properties of the equations of a model of coupled thermoplasticity, PMM, 36, 6, 1972.
5. DRUYANOV B.A., Generalized solutions of the dynamic theory of plasticity and thermoplasticity. Dokl. Akad. Nauk SSSR, 267, 5, 1982.
6. RANETSKII B. and SAVCHUK A., Temperature effects in plasticity. In books: Problems of the Theory of Plasticity and Creep. Moscow, Mir, 1979.
7. LIVSHITZ B.G., KRAPOSHIN V.S. and LINETSKII YA.P., Physical Properties of Metals and Alloys. Moscow, Metallurgiya, 1980.
8. DRUYANOV B.A., On strong discontinuities in compressible plastic media. Rheological models and processes of the deformation of porous powder and composite materials. Cool. of papers. Kiev, Naukova Dumka, 1985.
9. SEDOV L.I., Mechanics of a Continuous Medium. 1, Moscow, Nauka, 1976.
10. DRUYANOV B.A. and SVYATOVA E.A., On strong velocity discontinuities in thermoplastic media. Dokl. Akad. Nauk SSSR, 262, 2, 1982.

Translated by L.K.